

Feuille d'exercices 4

Exercice 1. Trouver toutes les solutions réelles des systèmes et équations suivants :

$$(1) \begin{cases} x'(t) = 2x(t) + 9y(t) + e^t, \\ y'(t) = x(t) + 2y(t), \end{cases}$$

$$(2) \begin{cases} x'(t) = 2x(t) + 9y(t), \\ y'(t) = x(t) + 2y(t) - \sin t, \end{cases}$$

$$(3) \begin{cases} x'(t) = 2x(t) + 9y(t) + \frac{3}{e^t + 1}, \\ y'(t) = x(t) + 2y(t) - \frac{1}{e^t + 1}, \end{cases}$$

$$(4) \begin{cases} x'(t) = x(t) - 4y(t) - e^t, \\ y'(t) = x(t) + y(t) + e^t, \end{cases}$$

$$(5) \begin{cases} x'(t) = x(t) - 4y(t) + e^t \cos t, \\ y'(t) = x(t) + y(t), \end{cases}$$

$$(6) \begin{cases} x'(t) = y(t) + \tan^2 t - 1, \\ y'(t) = -x(t) + \tan t. \end{cases}$$

$$(7) \begin{cases} x'(t) = x(t) - 2y(t) - z(t) + e^t, \\ y'(t) = -x(t) + y(t) + z(t), \\ z'(t) = x(t) - z(t) - e^t, \end{cases}$$

$$(8) \begin{cases} x'(t) = x(t) - y(t) - z(t), \\ y'(t) = x(t) + y(t) + \sin t, \\ z'(t) = 3x(t) + z(t). \end{cases}$$

$$(9) \quad x''(t) + 3x'(t) + 2x(t) = \frac{1}{e^t + 1},$$

$$(10) \quad x''(t) - x(t) = \frac{1}{t} - \frac{2}{t^3}.$$

Exercice 2. Dessiner les courbes définies par les équations suivantes.

1. $x^2 - y^2 = 9$,
2. $4x^2 - y^2 = 1$,
3. $x^2 + 4y^2 = 4$,
4. $x^2 - y^2 = x$,
5. $x^2 + y^2 = x + y$,
6. $14x^2 + 4xy + 11y^2 = 100$,
7. $x^2 + y^2 + 2x + 2y = 4xy$,
8. $x^2 + y^2 + 2x + 4y = 2xy$,
9. $7x^2 + y^2 = 4x + 4y + 8xy$.
10. $(x + y)^2 = 2x$.